

# Direct CP violation of $b \rightarrow s\gamma$ and CP asymmetries of non-leptonic $B$ decays in squark flavor mixing

Yusuke Shimizu<sup>1,\*</sup>, Morimitsu Tanimoto<sup>1,†</sup> and Kei Yamamoto<sup>2,‡</sup>

<sup>1</sup>*Department of Physics, Niigata University, Niigata 950-2181, Japan*

<sup>2</sup>*Graduate School of Science and Technology, Niigata University,  
Niigata 950-2181, Japan*

## Abstract

We study the contribution of the squark flavor mixing from the  $LR(RL)$  component of the squark mass matrices to the direct CP violation of the  $b \rightarrow s\gamma$  decay and the CP-violating asymmetry in the non-leptonic decays of  $B$  mesons. The magnitude of the  $LR(RL)$  component is constrained by the branching ratio and the direct CP violation of  $b \rightarrow s\gamma$ . We predict the correlation of the CP asymmetries among  $A_{\text{CP}}^{b \rightarrow s\gamma}$ ,  $\mathcal{S}_{\phi K_S}$  and  $\mathcal{S}_{\eta' K^0}$  of the  $B$  decays. The precise data of these CP violations will give us the crucial test for our framework of the squark flavor mixing.

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\*E-mail address: shimizu@muse.sc.niigata-u.ac.jp

†E-mail address: tanimoto@muse.sc.niigata-u.ac.jp

‡E-mail address: yamamoto@muse.sc.niigata-u.ac.jp

# 1 Introduction

New physics are expected to be observed at the LHC experiments. Although new particles have not yet discovered, LHCb has reported new data of the CP violation of  $B$  mesons and the branching ratios of rare  $B$  decays. New physics are also expected to be found in the  $B$  meson decays.

The CP violation in the  $K$  and  $B_d$  mesons has been successfully understood within the framework of the standard model (SM), so called Kobayashi-Maskawa (KM) model [1]. The source of the CP violation is the KM phase in the quark sector with three families. However, there could be new sources of the CP violation if the SM is extended to the supersymmetric (SUSY) models. The CP-violating phases appear in soft scalar mass matrices. These phases contribute to flavor changing neutral currents with the CP violation. Therefore, we expect the SUSY contribution in the CP-violating phenomena in the  $B$  meson decays.

The typical contribution of SUSY is the gluino-squark mediated flavor changing process [2]-[11]. In our previous paper [12], we have already discussed the effect of the squark flavor mixing on the CP violation in the non-leptonic decays of  $B_d^0$  and  $B_s^0$  taking account of the recent LHCb experimental data. We have found the deviation from the SM predictions in the asymmetries of the penguin dominated decays  $B_d^0 \rightarrow \phi K_S$  and  $B_d^0 \rightarrow \eta' K^0$ . In that framework of the SUSY contribution, we assume that  $LR$  and  $RL$  components of the squark mass matrices are neglected. The  $LL$  and  $RR$  components of squark mass matrices contribute considerably to the penguin processes for the case of large  $\mu \tan \beta$ ,  $\mathcal{O}(10 \text{ TeV})$ . However, if  $LR$  and  $RL$  components of squark mass matrices dominate the penguin decays, the asymmetries of  $B_d^0 \rightarrow \phi K_S$  and  $B_d^0 \rightarrow \eta' K^0$  are deviated from the SM predictions even for the case of the smaller  $\mu \tan \beta$ ,  $\mathcal{O}(1 \text{ TeV})$ . Then, these contributions of the new physics are correlated with the direct CP violation of the  $b \rightarrow s\gamma$  decay. In this paper, we present the numerical analyses in the case that  $LR$  and  $RL$  components of squark mass matrices dominate the penguin decays. In this case, the  $LR(RL)$  components do not contribute to the dispersive part  $M_{12}^q$  of  $B_q - \bar{B}_q$  ( $q = d, s$ ) mixing.

In section 2, we summarize the effect of new physics in the CP violations of the neutral  $B$  mesons including the recent experimental data. In section 3, we discuss the our framework of the squark flavor mixing in the CP violation of  $B$  mesons. We also discuss the constraints from the direct CP violation in the  $b \rightarrow s\gamma$  process. In section 4, we show the numerical result of the CP violation in the  $B$  mesons. Section 5 is devoted to the summary.

## 2 New physics of CP violation in $B$ mesons

Let us discuss the effect of new physics in the non-leptonic decays of  $B$  mesons. The contribution of new physics to the dispersive part  $M_{12}^q$  ( $q = d, s$ ) is parameterized as

$$M_{12}^q = M_{12}^{q,\text{SM}} + M_{12}^{q,\text{SUSY}} = M_{12}^{q,\text{SM}}(1 + h_q e^{2i\sigma_q}) , \quad (q = d, s) \quad (1)$$

where  $M_{12}^{q,\text{SUSY}}$  is the SUSY contribution, and the SM contribution  $M_{12}^{q,\text{SM}}$  is given as [13]

$$M_{12}^{q,\text{SM}} = \frac{G_F^2 M_{B_q}}{12\pi^2} M_W^2 (V_{tb} V_{tq}^*)^2 \hat{\eta}_B S_0(x_t) f_{B_q}^2 B_q . \quad (2)$$

The time dependent CP asymmetry decaying into the final state  $f$  is defined as [14]

$$\mathcal{S}_f = \frac{2\text{Im}\lambda_f}{|\lambda_f|^2 + 1}, \quad (3)$$

where

$$\lambda_f = \frac{q}{p}\bar{\rho}, \quad \frac{q}{p} = \sqrt{\frac{M_{12}^{q*} - \frac{i}{2}\Gamma_{12}^*}{M_{12}^q - \frac{i}{2}\Gamma_{12}}}, \quad \bar{\rho} \equiv \frac{\bar{A}(\bar{B}_q^0 \rightarrow f)}{A(B_q^0 \rightarrow f)}. \quad (4)$$

In the  $B_d^0 \rightarrow J/\psi K_S$  decay,  $\lambda_{J/\psi K_S}$  is given in terms of the new physics parameters  $h_d$  and  $\sigma_d$  as

$$\lambda_{J/\psi K_S} = -e^{-i\phi_d}, \quad \phi_d = 2\beta_d + \arg(1 + h_d e^{2i\sigma_d}), \quad (5)$$

by putting  $|\bar{\rho}| = 1$  and  $q/p \simeq \sqrt{M_{12}^{q*}/M_{12}^q}$ , where the phase  $\beta_d$  is given in the SM. The CKMfitter provided the allowed region of  $h_d$  and  $\sigma_d$ , where the central values are [15, 16]

$$h_d \simeq 0.3, \quad \sigma_d \simeq 1.8 \text{ rad}. \quad (6)$$

Since penguin processes are dominant in the case of  $f = \phi K_S, \eta' K^0$ , the loop induced new physics could contribute considerably on the CP violation of those decays. Then,  $\mathcal{S}_f$  is not any more same as  $\mathcal{S}_{J/\psi K_S}$  if new physics leads to  $|\bar{\rho}| \neq 1$ . Those predictions provide us good tests for new physics.

In the  $B_s^0 \rightarrow J/\psi \phi$  decay, we parametrize as

$$\lambda_{J/\psi \phi} = e^{-i\phi_s}, \quad \phi_s = -2\beta_s + \arg(1 + h_s e^{2i\sigma_s}), \quad (7)$$

where  $\beta_s$  is given in the SM. Recently the LHCb presented the observed CP-violating phase  $\phi_s$  in  $\bar{B}_s^0 \rightarrow J/\psi \pi^+ \pi^-$  decay using about  $1 \text{ fb}^{-1}$  of data [17]. This result leads to

$$\phi_s = -0.019_{-0.174-0.03}^{+0.173+0.04} \text{ rad}, \quad (8)$$

which is consistent with the SM prediction [15]

$$\phi_s^{J/\psi \phi, SM} = -2\beta_s = -0.0363 \pm 0.0017 \text{ rad}. \quad (9)$$

Taking account of these data, the CKMfitter has presented the allowed values of  $h_s$  and  $\sigma_s$  [15, 16]. The allowed region is rather large including zero values. In order to investigate possible contribution of new physics, we take the central values

$$h_s = 0.1, \quad \sigma_s = 0.9 - 2.2 \text{ rad}, \quad (10)$$

as a typical parameter set in our work.

We remark on numerical inputs of phases  $\phi_d$  and  $\phi_s$  in our calculation. The phase  $\phi_d$  is derived from the observed value  $\mathcal{S}_f = 0.671 \pm 0.023$  in  $B_d^0 \rightarrow J/\psi K_S$  [18] as seen Eqs.(3) and (5). On the other hand, we use the SM value of  $\beta_s$  and the values of the new physics parameters,  $h_s$  and  $\sigma_s$  in Eq.(10) to estimate  $\phi_s = -2\beta_s + \arg(1 + h_s e^{2i\sigma_s})$ . We do not use the observed value of  $\phi_s$  in  $B_s^0 \rightarrow J/\psi \phi$  because of the large experimental error in Eq.(8).

Since the  $B_d^0 \rightarrow J/\psi K_S$  process occurs at the tree level in SM, the CP-violating asymmetry originates from  $M_{12}^d$ . Although the  $B_d^0 \rightarrow \phi K_S$  and  $B_d^0 \rightarrow \eta' K^0$  decays are penguin dominant ones, their asymmetries also come from  $M_{12}^d$ . Then, asymmetries of  $B_d^0 \rightarrow J/\psi K_S$ ,  $B_d^0 \rightarrow \phi K_S$  and  $B_d^0 \rightarrow \eta' K^0$  are expected to be same magnitude in SM.

On the other hand, if the squark flavor mixing contributes to the decay at the one-loop level, its magnitude could be comparable to the SM penguin one in  $B_d^0 \rightarrow \phi K_S$  and  $B_d^0 \rightarrow \eta' K^0$ , but it is tiny in  $B_d^0 \rightarrow J/\psi K_S$ . Endo, Mishima and Yamaguchi proposed the possibility to find the SUSY contribution in these asymmetries [19]. The present data suggest the deviation from SM in these time dependent asymmetries of  $B_d^0$  decays such as,

$$\mathcal{S}_{J/\psi K_S} = 0.671 \pm 0.023, \quad \mathcal{S}_{\phi K_S} = 0.39 \pm 0.17, \quad \mathcal{S}_{\eta' K^0} = 0.60 \pm 0.07, \quad (11)$$

however, precise data are required to justify the new physics contribution.

New physics contribute to the  $b \rightarrow s\gamma$  process. The observed  $b \rightarrow s\gamma$  branching ratio (BR) is  $(3.60 \pm 0.23) \times 10^{-4}$  [18], on the other hand the SM prediction is given as  $(3.15 \pm 0.23) \times 10^{-4}$  at  $\mathcal{O}(\alpha_s^2)$  [20, 21]. Therefore, the contribution of new physics should be suppressed. New physics are also constrained by the direct CP violation

$$A_{\text{CP}}^{b \rightarrow s\gamma} \equiv \frac{\Gamma(\bar{B} \rightarrow X_s \gamma) - \Gamma(B \rightarrow X_{\bar{s}} \gamma)}{\Gamma(\bar{B} \rightarrow X_s \gamma) + \Gamma(B \rightarrow X_{\bar{s}} \gamma)}. \quad (12)$$

Since SM prediction  $A_{\text{CP}}^{b \rightarrow s\gamma} \simeq 0.005$  is tiny [22], new physics may appear in this CP asymmetry. The present data  $A_{\text{CP}}^{b \rightarrow s\gamma} = -0.008 \pm 0.029$  [18] has rather large error bar, and so the constraint of new physics is not so severe. However, improved data will provide the crucial test for new physics.

### 3 Squark flavor mixing and CP violations of $B$ mesons

Let us consider the flavor structure of squarks in order to estimate the CP-violating asymmetries of  $B$  meson decays. We take the most popular ansatz, which is to postulate a degenerate SUSY breaking mass spectrum for down-type squarks. Then, in the super-CKM basis, we can parametrize the soft scalar masses squared  $M_{d_{LL}}^2$ ,  $M_{d_{RR}}^2$ ,  $M_{d_{LR}}^2$ , and  $M_{d_{RL}}^2$  for the down-type squarks as follows:

$$\begin{aligned} M_{d_{LL}}^2 &= m_{\tilde{q}}^2 \begin{pmatrix} 1 + (\delta_d^{LL})_{11} & (\delta_d^{LL})_{12} & (\delta_d^{LL})_{13} \\ (\delta_d^{LL})_{12}^* & 1 + (\delta_d^{LL})_{22} & (\delta_d^{LL})_{23} \\ (\delta_d^{LL})_{13}^* & (\delta_d^{LL})_{23}^* & 1 + (\delta_d^{LL})_{33} \end{pmatrix}, \\ M_{d_{RR}}^2 &= m_{\tilde{q}}^2 \begin{pmatrix} 1 + (\delta_d^{RR})_{11} & (\delta_d^{RR})_{12} & (\delta_d^{RR})_{13} \\ (\delta_d^{RR})_{12}^* & 1 + (\delta_d^{RR})_{22} & (\delta_d^{RR})_{23} \\ (\delta_d^{RR})_{13}^* & (\delta_d^{RR})_{23}^* & 1 + (\delta_d^{RR})_{33} \end{pmatrix}, \\ M_{d_{LR}}^2 &= (M_{d_{RL}}^2)^\dagger = m_{\tilde{q}}^2 \begin{pmatrix} (\delta_d^{LR})_{11} & (\delta_d^{LR})_{12} & (\delta_d^{LR})_{13} \\ (\delta_d^{LR})_{21} & (\delta_d^{LR})_{22} & (\delta_d^{LR})_{23} \\ (\delta_d^{LR})_{31} & (\delta_d^{LR})_{32} & (\delta_d^{LR})_{33} \end{pmatrix}, \end{aligned} \quad (13)$$

where  $m_{\tilde{q}}$  is the average squark mass, and  $(\delta_d^{LL})_{ij}$ ,  $(\delta_d^{LR})_{ij}$ ,  $(\delta_d^{RL})_{ij}$ , and  $(\delta_d^{RR})_{ij}$  are called as the mass insertion (MI) parameters. The MI parameters are supposed to be much smaller than 1.

The contribution of the gluino-squark box diagram to the dispersive part of the effective Hamiltonian for the  $B_q$ - $\bar{B}_q$  mixing is written as [23, 24]

$$M_{12}^{q,SUSY} = A_1^q \left[ A_2 \{ (\delta_d^{LL})_{ij}^2 + (\delta_d^{RR})_{ij}^2 \} + A_3^q (\delta_d^{LL})_{ij} (\delta_d^{RR})_{ij} \right. \\ \left. + A_4^q \{ (\delta_d^{LR})_{ij}^2 + (\delta_d^{RL})_{ij}^2 \} + A_5^q (\delta_d^{LR})_{ij} (\delta_d^{RL})_{ij} \right], \quad (14)$$

where

$$A_1^q = -\frac{\alpha_S^2}{216m_{\tilde{q}}^2} \frac{2}{3} M_{B_q} f_{B_q}^2, \quad A_2 = 24x f_6(x) + 66\tilde{f}_6(x), \\ A_3^q = \left\{ 384 \left( \frac{M_{B_q}}{m_j + m_i} \right)^2 + 72 \right\} x f_6(x) + \left\{ -24 \left( \frac{M_{B_q}}{m_j + m_i} \right)^2 + 36 \right\} \tilde{f}_6(x), \\ A_4^q = \left\{ -132 \left( \frac{M_{B_q}}{m_j + m_i} \right)^2 \right\} x f_6(x), \quad A_5^q = \left\{ -144 \left( \frac{M_{B_q}}{m_j + m_i} \right)^2 - 84 \right\} \tilde{f}_6(x). \quad (15)$$

Here, we use  $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2$ , where  $m_{\tilde{g}}$  is the gluino mass. For the cases of  $q = d$  and  $q = s$ , we take  $(i, j) = (1, 3)$  and  $(i, j) = (2, 3)$ , respectively, where  $m_1 = m_d$ ,  $m_2 = m_s$  and  $m_3 = m_b$ . The loop functions  $f_6(x)$  and  $\tilde{f}_6(x)$  are shown in [12].

For the case of  $x \simeq 1$ , we get  $A_2 \simeq -1$ ,  $A_3^q \simeq 30$ ,  $A_4^q \simeq -10$  and  $A_5^q \simeq 10$ . Therefore, each term at the r.h.s. of Eq.(14) may contribute to  $M_{12}^{q,SUSY}$  comparably. However, magnitudes of  $(\delta_d^{LR})_{ij}$  and  $(\delta_d^{RL})_{ij}$  are constrained severely by the  $b \rightarrow s\gamma$  decay as discussed later.

The squark flavor mixing can be tested in the CP-violating asymmetries in the neutral  $B$  meson decays. Let us present the framework of these calculations. The effective Hamiltonian for  $\Delta B = 1$  process is defined as

$$H_{eff} = \frac{4G_F}{\sqrt{2}} \left[ \sum_{q'=u,c} V_{q'b} V_{q's}^* \sum_{i=1,2} C_i O_i^{(q')} - V_{tb} V_{ts}^* \sum_{i=3-6,7\gamma,8G} \left( C_i O_i + \tilde{C}_i \tilde{O}_i \right) \right], \quad (16)$$

where the local operators are given as

$$O_1^{(q')} = (\bar{s}_\alpha \gamma_\mu P_L q'_\beta) (\bar{q}'_\beta \gamma^\mu P_L b_\alpha), \quad O_2^{(q')} = (\bar{s}_\alpha \gamma_\mu P_L q'_\alpha) (\bar{q}'_\beta \gamma^\mu P_L b_\beta), \\ O_3 = (\bar{s}_\alpha \gamma_\mu P_L b_\alpha) \sum_q (\bar{q}_\beta \gamma^\mu P_L q_\beta), \quad O_4 = (\bar{s}_\alpha \gamma_\mu P_L b_\beta) \sum_q (\bar{q}_\beta \gamma^\mu P_L q_\alpha), \\ O_5 = (\bar{s}_\alpha \gamma_\mu P_L b_\alpha) \sum_q (\bar{q}_\beta \gamma^\mu P_R q_\beta), \quad O_6 = (\bar{s}_\alpha \gamma_\mu P_L b_\beta) \sum_q (\bar{q}_\beta \gamma^\mu P_R q_\alpha), \\ O_{7\gamma} = \frac{e}{16\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu} P_R b_\alpha F_{\mu\nu}, \quad O_{8G} = \frac{g_s}{16\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu} P_R T_{\alpha\beta}^a b_\beta G_{\mu\nu}^a, \quad (17)$$

where  $P_R = (1 + \gamma_5)/2$ ,  $P_L = (1 - \gamma_5)/2$ , and  $\alpha$  and  $\beta$  are color indices, and  $q$  is taken to be  $u, d, s, c$ . Here,  $C_i$ 's  $\tilde{C}_i$ 's are the Wilson coefficients, and  $\tilde{O}_i$ 's are the operators by replacing

$L(R)$  with  $R(L)$  in  $O_i$ . In this paper,  $C_i$  includes both SM contribution and gluino one, such as  $C_i = C_i^{\text{SM}} + C_i^{\tilde{g}}$ , where  $C_i^{\text{SM}}$  is given in Ref. [25] and  $C_i^{\tilde{g}}$  is presented as follows [26]:

$$\begin{aligned}
C_3^{\tilde{g}} &\simeq \frac{\sqrt{2}\alpha_s^2}{4G_F V_{tb} V_{ts}^* m_{\tilde{q}}^2} (\delta_d^{LL})_{23} \left[ -\frac{1}{9}B_1(x) - \frac{5}{9}B_2(x) - \frac{1}{18}P_1(x) - \frac{1}{2}P_2(x) \right], \\
C_4^{\tilde{g}} &\simeq \frac{\sqrt{2}\alpha_s^2}{4G_F V_{tb} V_{ts}^* m_{\tilde{q}}^2} (\delta_d^{LL})_{23} \left[ -\frac{7}{3}B_1(x) + \frac{1}{3}B_2(x) + \frac{1}{6}P_1(x) + \frac{3}{2}P_2(x) \right], \\
C_5^{\tilde{g}} &\simeq \frac{\sqrt{2}\alpha_s^2}{4G_F V_{tb} V_{ts}^* m_{\tilde{q}}^2} (\delta_d^{LL})_{23} \left[ \frac{10}{9}B_1(x) + \frac{1}{18}B_2(x) - \frac{1}{18}P_1(x) - \frac{1}{2}P_2(x) \right], \\
C_6^{\tilde{g}} &\simeq \frac{\sqrt{2}\alpha_s^2}{4G_F V_{tb} V_{ts}^* m_{\tilde{q}}^2} (\delta_d^{LL})_{23} \left[ -\frac{2}{3}B_1(x) + \frac{7}{6}B_2(x) + \frac{1}{6}P_1(x) + \frac{3}{2}P_2(x) \right], \\
C_{7\gamma}^{\tilde{g}} &\simeq -\frac{\sqrt{2}\alpha_s\pi}{6G_F V_{tb} V_{ts}^* m_{\tilde{q}}^2} \left[ (\delta_d^{LL})_{23} \left( \frac{8}{3}M_3(x) - \mu \tan \beta \frac{m_{\tilde{g}}}{m_{\tilde{q}}^2} \frac{8}{3}M_a(x) \right) + (\delta_d^{LR})_{23} \frac{m_{\tilde{g}}}{m_b} \frac{8}{3}M_1(x) \right], \\
C_{8G}^{\tilde{g}} &\simeq -\frac{\sqrt{2}\alpha_s\pi}{2G_F V_{tb} V_{ts}^* m_{\tilde{q}}^2} \left[ (\delta_d^{LL})_{23} \left\{ \left( \frac{1}{3}M_3(x) + 3M_4(x) \right) \right. \right. \\
&\quad \left. \left. - \mu \tan \beta \frac{m_{\tilde{g}}}{m_{\tilde{q}}^2} \left( \frac{1}{3}M_a(x) + 3M_b(x) \right) \right\} + (\delta_d^{LR})_{23} \frac{m_{\tilde{g}}}{m_b} \left( \frac{1}{3}M_1(x) + 3M_2(x) \right) \right]. \quad (18)
\end{aligned}$$

Here the double mass insertion is included in  $C_{7\gamma}^{\tilde{g}}$  and  $C_{8G}^{\tilde{g}}$ . The Wilson coefficients  $\tilde{C}_i^{\tilde{g}}$ 's are obtained by replacing  $L(R)$  with  $R(L)$  in  $C_i^{\tilde{g}}$ 's. The loop functions, which we use in our calculations, are presented in our previous paper [12].

The CP-violating asymmetries  $\mathcal{S}_f$  in Eq. (3) are calculated by using  $\lambda_f$ , which is given for  $B_d^0 \rightarrow \phi K_S$  and  $B_d^0 \rightarrow \eta' K^0$  as follows:

$$\lambda_{\phi K_S, \eta' K^0} = -e^{-i\phi_d} \frac{\sum_{i=3-6,7\gamma,8G} \left( C_i^{\text{SM}} \langle O_i \rangle + C_i^{\tilde{g}} \langle O_i \rangle + \tilde{C}_i^{\tilde{g}} \langle \tilde{O}_i \rangle \right)}{\sum_{i=3-6,7\gamma,8G} \left( C_i^{\text{SM}*} \langle O_i \rangle + C_i^{\tilde{g}*} \langle O_i \rangle + \tilde{C}_i^{\tilde{g}*} \langle \tilde{O}_i \rangle \right)}, \quad (19)$$

where  $\langle O_i \rangle$  is the abbreviation of  $\langle f | O_i | B_d^0 \rangle$ . It is noticed that  $\langle \phi K_S | O_i | B_d^0 \rangle = \langle \phi K_S | \tilde{O}_i | B_d^0 \rangle$  and  $\langle \eta' K^0 | O_i | B_d^0 \rangle = -\langle \eta' K^0 | \tilde{O}_i | B_d^0 \rangle$  because of the parity of the final state. We have also  $\lambda_f$  for  $B_s^0 \rightarrow \phi\phi$  as follow:

$$\lambda_{\phi\phi} = e^{-i\phi_s} \frac{\sum_{i=3-6,7\gamma,8G} C_i^{\text{SM}} \langle O_i \rangle + C_i^{\tilde{g}} \langle O_i \rangle + \tilde{C}_i^{\tilde{g}} \langle \tilde{O}_i \rangle}{\sum_{i=3-6,7\gamma,8G} C_i^{\text{SM}*} \langle O_i \rangle + C_i^{\tilde{g}*} \langle O_i \rangle + \tilde{C}_i^{\tilde{g}*} \langle \tilde{O}_i \rangle}, \quad (20)$$

with  $\langle \phi\phi | O_i | B_s^0 \rangle = -\langle \phi\phi | \tilde{O}_i | B_s^0 \rangle$ .

In these non-leptonic decays, the  $C_{8G}^{\tilde{g}} \langle O_{8G} \rangle$  dominates these amplitude, but small contributions from other Wilson coefficients are also taken account in our calculations. Therefore, we estimate each hadronic matrix elements by using the factorization relations in Ref. [27].

Let us discuss the each contribution of the mass insertion parameters to  $C_{8G}^{\tilde{g}}$  in Eq.(18). Taking account that the loop functions  $M_i(x)$  are of same order and  $m_{\tilde{g}} \simeq m_{\tilde{q}}$ , the ratio of  $LL$  component and  $LR$  one is  $(\delta_d^{LL})_{23} \times \mu \tan \beta / m_{\tilde{q}}$  to  $(\delta_d^{LR})_{23} \times m_{\tilde{q}} / m_b$ . If  $\mathcal{O}(\mu \tan \beta) \simeq \mathcal{O}(m_{\tilde{q}})$  and  $m_{\tilde{q}} \geq 1$  TeV, the  $LR$  component may contribute significantly to  $C_{8G}^{\tilde{g}}$  due to the enhancement factor  $m_{\tilde{q}} / m_b = \mathcal{O}(10^2)$ . For example, in the case of  $(\delta_d^{LL})_{23} = 10^{-2}$  and  $(\delta_d^{LR})_{23} = 10^{-3}$ , the  $LR$  component dominate  $C_{8G}^{\tilde{g}}$ , while it is minor in  $M_{12}^{q, \text{SUSY}}$  as seen in Eq.(14). This situation is also kept in the  $b \rightarrow s\gamma$  decay.

The  $b \rightarrow s\gamma$  decay is a typical one to investigate new physics. The branching ratio is given as

$$\frac{BR(B \rightarrow X_s \gamma)}{BR(B \rightarrow X_c e \bar{\nu}_e)} = \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} \frac{6\alpha}{\pi f(z)} |C_{7\gamma}^{\text{eff}}|^2, \quad (21)$$

where

$$f(z) = 1 - 8z + 8z^3 - z^4 - 12z^2 \ln z, \quad z = \frac{m_c^2}{m_b^2}. \quad (22)$$

Here,  $C_{7\gamma}^{\text{eff}}$  includes both contributions from the SM and the gluino-squark flavor mixing  $C_{7\gamma}^{\tilde{g}}$ . As seen in Eq.(18), both  $C_{7\gamma}^{\tilde{g}}$  and  $C_{8G}^{\tilde{g}}$  have the similar dependence of  $(\delta_d^{LR})_{23}$ . Therefore, we should discuss carefully the contribution from  $(\delta_d^{LR})_{23}$  in our numerical calculations.

We can discuss the direct CP violation  $A_{\text{CP}}^{b \rightarrow s\gamma}$  in the  $b \rightarrow s\gamma$  decay, which is given as [22]

$$\begin{aligned} A_{\text{CP}}^{b \rightarrow s\gamma} &= \frac{\Gamma(\bar{B} \rightarrow X_s \gamma) - \Gamma(B \rightarrow X_{\bar{s}} \gamma)}{\Gamma(\bar{B} \rightarrow X_s \gamma) + \Gamma(B \rightarrow X_{\bar{s}} \gamma)} \Big|_{E_\gamma > (1-\delta)E_\gamma^{\text{max}}} \\ &= \frac{\alpha_s(m_b)}{|C_{7\gamma}|^2} \left[ \frac{40}{81} \text{Im}[C_2 C_{7\gamma}^*] - \frac{8z}{9} [v(z) + b(z, \delta)] \text{Im} \left[ \left( 1 + \frac{V_{us}^* V_{ub}}{V_{ts}^* V_{tb}} \right) C_2 C_{7\gamma}^* \right] \right. \\ &\quad \left. - \frac{4}{9} \text{Im}[C_{8G} C_{7\gamma}^*] + \frac{8z}{27} b(z, \delta) \text{Im} \left[ \left( 1 + \frac{V_{us}^* V_{ub}}{V_{ts}^* V_{tb}} \right) C_2 C_{8G}^* \right] \right], \end{aligned}$$

where  $v(z)$  and  $b(z, \delta)$  are explicitly given in [22], and  $C_i^{\text{eff}}$  includes both the SM and SUSY contributions. Although the experimental data has still large error bar, we can discuss the SUSY contribution to  $A_{\text{CP}}^{b \rightarrow s\gamma}$ .

Let us set up the framework of our calculations. Suppose that  $\mu \tan \beta$  is at most  $\mathcal{O}(1)$  TeV. Then, magnitudes of  $(\delta_d^{LL})_{23}$  and  $(\delta_d^{RR})_{23}$  are constrained by  $M_{12}^s$  as seen in Eq.(14). Taking account of  $h_s = 0.1$  in Eq.(10), we obtain  $|(\delta_d^{LL})_{23}| \simeq |(\delta_d^{RR})_{23}| \simeq 0.02$  in our previous work [12]. Then, these contributions to  $C_{7\gamma}^{\tilde{g}}$  and  $C_{8G}^{\tilde{g}}$  are minor.

On the other hand,  $(\delta_d^{LR})_{23}$  and  $(\delta_d^{RL})_{23}$  are severely constrained by  $C_{7\gamma}^{\text{eff}}$  and  $C_{8G}^{\text{eff}}$ . We show the constraint for  $(\delta_d^{LR})_{23}$  and  $(\delta_d^{RL})_{23}$  in our following calculations. In our convenience, we suppose  $|(\delta_d^{LR})_{23}| = |(\delta_d^{RL})_{23}|$ . Then, we can parametrize these parameters as follows:

$$(\delta_d^{LR})_{23} = |(\delta_d^{LR})_{23}| e^{2i\theta_{23}^{LR}}, \quad (\delta_d^{RL})_{23} = |(\delta_d^{LR})_{23}| e^{2i\theta_{23}^{RL}}, \quad (23)$$

where  $\theta_{23}^{LR}$  and  $\theta_{23}^{RL}$  are taken in the region  $[0 - \pi]$ . By using this set up, we show numerical analyses in the next section.



## 4 Numerical analyses

In this section, we show the numerical analyses of the CP-violation in the  $B$  mesons. In our following numerical calculations, we fix the squark mass and the gluino mass as

$$m_{\tilde{q}} = 1000 \text{ GeV}, \quad m_{\tilde{g}} = 1500 \text{ GeV}, \quad (24)$$

which are consistent with recent lower bound of these masses at LHC [28]. We use relevant parameters as given in [12] to estimate the SM contribution.

At first, we discuss the  $b \rightarrow s\gamma$  decay. The observed  $b \rightarrow s\gamma$  branching ratio is  $(3.60 \pm 0.23) \times 10^{-4}$  [18], on the other hand the SM prediction is given as  $(3.15 \pm 0.23) \times 10^{-4}$  at  $\mathcal{O}(\alpha_s^2)$  [20, 21]. Since  $\mu \tan \beta$  is supposed to be lower than  $\mathcal{O}(1 \text{ TeV})$ , the contribution of  $(\delta_d^{LL})_{23} \simeq (\delta_d^{RR})_{23}$  is negligibly small. The contribution of  $(\delta_d^{LR})_{23}$  becomes important through the interference with the SM component in the decay amplitude. On the other hand, since  $(\delta_d^{RL})_{23}$  does not interfere with the SM component, its contribution is minor. The branching ratio gives the constraint for the magnitude of  $(\delta_d^{LR})_{23}$ . The direct CP violation of the  $b \rightarrow s\gamma$  is also useful to constraint  $(\delta_d^{LR})_{23}$ . We show the  $A_{\text{CP}}^{b \rightarrow s\gamma}$  versus  $|(\delta_d^{LR})_{23}|$  in Figure 1, where the upper and lower bounds of the experimental data with 90% C.L. are denoted red lines, and the predicted value of the SM is shown by the green line as the eye guide. As far as  $|(\delta_d^{LR})_{23}| \leq 10^{-3}$ , the predicted value is within the experimental allowed region.

In Figure 2, we show the  $|(\delta_d^{LR})_{23}|$  dependence of the branching ratio taking account the constraint of  $A_{\text{CP}}^{b \rightarrow s\gamma}$  as seen in Figure 1. Here, the allowed region at  $|(\delta_d^{LR})_{23}| = 0$  is the SM prediction. As the magnitude of  $(\delta_d^{LR})_{23}$  increases, the predicted region of the branching ratio splits into the larger region and smaller one. The excluded region between two regions is due to the constraint of  $A_{\text{CP}}^{b \rightarrow s\gamma}$ . Then, the predicted branching ratio becomes inconsistent with the experimental data at  $|(\delta_d^{LR})_{23}| \geq 5.5 \times 10^{-3}$ .

In order to see the role of the phase  $\theta_{23}^{LR}$ , we show  $A_{\text{CP}}^{b \rightarrow s\gamma}$  versus  $\theta_{23}^{LR}$  for  $|(\delta_d^{LR})_{23}| = 10^{-3}$  (blue) and  $|(\delta_d^{LR})_{23}| = 10^{-4}$  (orange) in Figure 3. The pink horizontal lines denote the

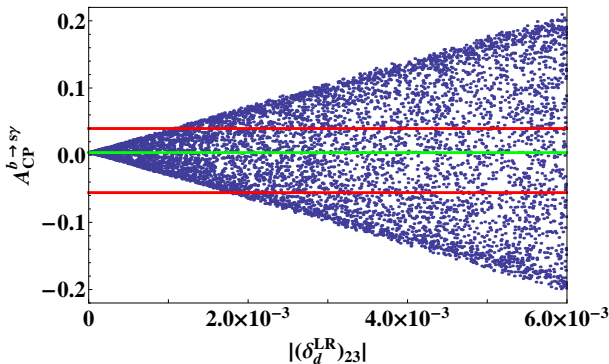


Figure 1: The direct CP violation  $A_{\text{CP}}^{b \rightarrow s\gamma}$  versus  $|(\delta_d^{LR})_{23}|$ , where the green line denotes the SM prediction and red lines denote the upper and lower bounds of the experimental data with 90% C.L..

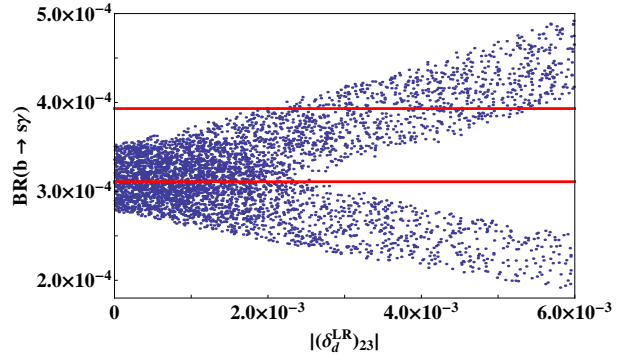


Figure 2: The predicted branching ratio of  $b \rightarrow s\gamma$  versus  $|(\delta_d^{LR})_{23}|$ , where the experimental constraint of  $A_{\text{CP}}^{b \rightarrow s\gamma}$  is taken account. Predicted value at  $|(\delta_d^{LR})_{23}| = 0$  is the SM one.



experimental upper and lower bounds at  $1\sigma$  level. As seen in this figure, we find that the reduction of the experimental error-bar will constrain the SUSY phase  $\theta_{23}^{LR}$  severely.

In Figure 4, we plot the allowed region on the  $|(\delta_d^{LR})_{23}| - \theta_{23}^{LR}$  plane by putting the experimental data at 90% C.L. of the branching ratio and the direct CP violation  $A_{CP}^{b \rightarrow s\gamma}$ . The allowed region of  $|(\delta_d^{LR})_{23}|$  is cut at  $5.5 \times 10^{-3}$ , where  $\theta_{23}^{LR}$  is tuned around  $\pi/2$ . Around  $\pi/4$  and  $3\pi/4$ ,  $A_{CP}^{b \rightarrow s\gamma}$  give the severe constraint as seen in Figure 3. This CP violation phase also contributes on the CP-violating asymmetry of the non-leptonic decays of  $B_d^0$  and  $B_s^0$  mesons.

Let us discuss  $\mathcal{S}_f$ , which is the measure of the CP-violating asymmetry, for  $B_d^0 \rightarrow J/\psi K_S$ ,  $\phi K_S$ ,  $\eta' K^0$ . As discussed in Section 2, these  $\mathcal{S}_f$ 's are predicted to be same ones in the SM. On the other hand, if the squark flavor mixing contributes to the decay process at the one-loop level, these asymmetries are different from among as seen in Eq.(19).

Since the phase  $\theta_{23}^{LR}$  contributes to  $A_{CP}^{b \rightarrow s\gamma}$ ,  $\mathcal{S}_{\phi K_S}$  and  $\mathcal{S}_{\eta' K^0}$  of  $B_d^0$  decays. We expect the

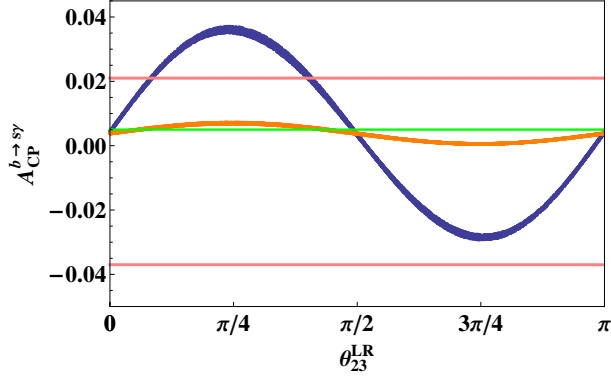


Figure 3:  $A_{CP}^{b \rightarrow s\gamma}$  versus  $\theta_{23}^{LR}$  for  $|(\delta_d^{LR})_{23}| = 10^{-3}$ (blue) and  $10^{-4}$ (orange), where the pink lines denote the experimental upper and lower bounds at  $1\sigma$  level.

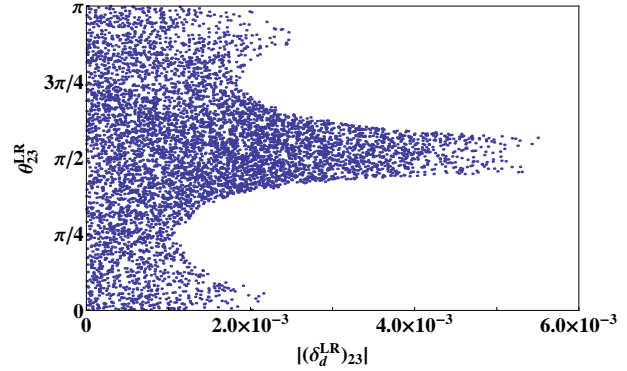


Figure 4: The allowed region on  $|(\delta_d^{LR})_{23}| - \theta_{23}^{LR}$  plane. The experimental data at 90% C.L. of the branching ratio and  $A_{CP}^{b \rightarrow s\gamma}$  are taken account.

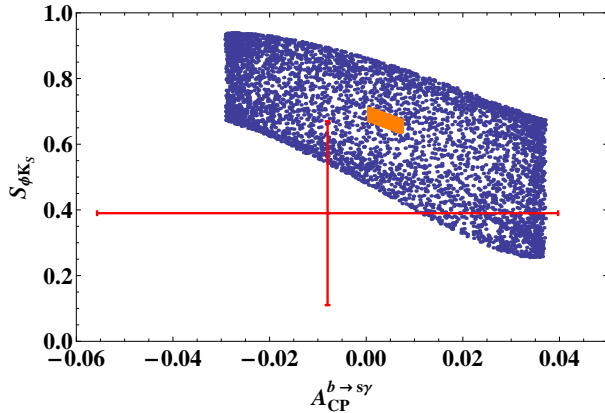


Figure 5:  $\mathcal{S}_{\phi K_S}$  of  $B_d^0$  versus  $A_{CP}^{b \rightarrow s\gamma}$  for  $|(\delta_d^{LR})_{23}| = 10^{-3}$ (blue) and  $10^{-4}$ (orange).

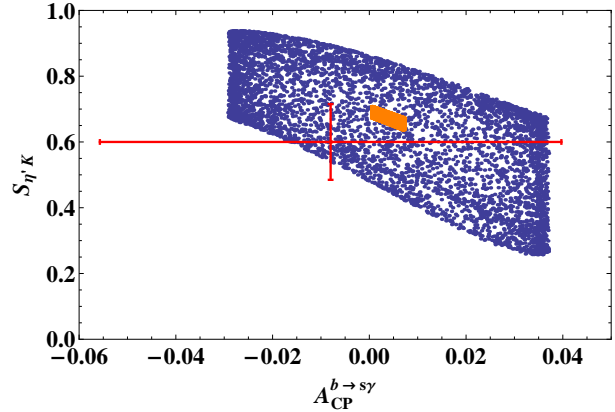


Figure 6:  $\mathcal{S}_{\eta' K^0}$  of  $B_d^0$  versus  $A_{CP}^{b \rightarrow s\gamma}$  for  $|(\delta_d^{LR})_{23}| = 10^{-3}$ (blue) and  $10^{-4}$ (orange).

correlations among them. We fix  $|(\delta_d^{LR})_{23}| = 10^{-4}$  (orange) and  $10^{-3}$  (blue) for typical values in the following calculations. we show the predicted regions on the  $A_{\text{CP}}^{b \rightarrow s\gamma}$ - $\mathcal{S}_{\phi K_S}$  and  $A_{\text{CP}}^{b \rightarrow s\gamma}$ - $\mathcal{S}_{\eta' K^0}$  planes in Figures 5 and 6, respectively. The experimental data is denoted by red lines at 90% C.L.. We also present the predicted region on the  $\mathcal{S}_{\phi K_S}$ - $\mathcal{S}_{\eta' K^0}$  plane in Figure 7, the slant dashed line denotes the SM prediction  $\mathcal{S}_{J/\psi K_S} = \mathcal{S}_{\phi K_S} = \mathcal{S}_{\eta' K}$ , where the observed value  $\mathcal{S}_{J/\psi K_S} = 0.671 \pm 0.023$  is put. The reduction of the experimental error of  $A_{\text{CP}}^{b \rightarrow s\gamma}$  will give us severe predictions for  $\mathcal{S}_{\phi K_S}$  and  $\mathcal{S}_{\eta' K^0}$ . It is noticed that this predicted region is different from the one in the previous work [12], where  $(\delta_d^{LR})_{23}$  is neglected and  $\mu \tan \beta = \mathcal{O}(10 \text{ TeV})$ .

As seen in Figures 5, 6 and 7, the reduction of the experimental errors will provide powerful tool to find the contribution of the squark flavor mixing. At last, we show the correlation between  $A_{\text{CP}}^{b \rightarrow s\gamma}$  and  $\mathcal{S}_{\phi\phi}$  of the  $B_s^0$  decay in Figure 8. We expect the observation of the CP violation in  $B_s^0 \rightarrow \phi\phi$  at LHCb.

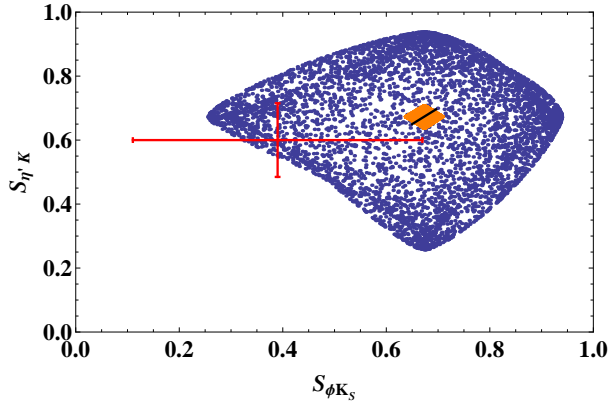


Figure 7: Predicted region on the  $\mathcal{S}_{\phi K_S}$ - $\mathcal{S}_{\eta' K^0}$  plane, where the slant dashed line denotes the SM prediction.

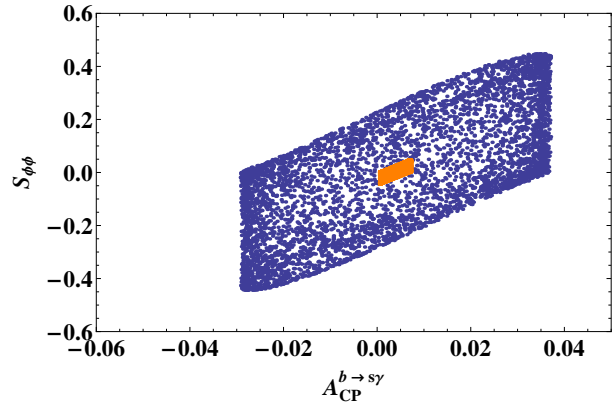


Figure 8: Predicted asymmetry  $\mathcal{S}_{\phi\phi}$  in the  $B_s^0$  decay versus  $A_{\text{CP}}^{b \rightarrow s\gamma}$ .

## 5 Summary

The CP violation of the neutral  $B$  meson is the important phenomenon to search for new physics. We have discussed the contribution of the squark flavor mixing from  $(\delta_d^{LR})_{23}$  and  $(\delta_d^{RL})_{23}$  on the direct CP violation of the  $b \rightarrow s\gamma$  decay and the CP-violating asymmetry in the non-leptonic decays of  $B_d^0$  and  $B_s^0$  mesons.

The magnitude of  $|(\delta_d^{LR})_{23}|$  is bounded by the branching ratio of  $b \rightarrow s\gamma$  with the constraint of  $A_{\text{CP}}^{b \rightarrow s\gamma}$ . The predicted branching ratio becomes inconsistent with the experimental data at  $|(\delta_d^{LR})_{23}| \geq 5.5 \times 10^{-3}$ . We have obtained the allowed region on the  $|(\delta_d^{LR})_{23}|$ - $\theta_{23}^{LR}$  plane. While the  $|(\delta_d^{LR})_{23}|$  is cut at  $5.5 \times 10^{-3}$ , CP-violating phase  $\theta_{23}^{LR}$  is severely constrained at  $|(\delta_d^{LR})_{23}| \geq 2 \times 10^{-3}$ . This CP-violating phase also contribute to the CP-violating asymmetry in the non-leptonic decays of  $B_d^0$  and  $B_s^0$  mesons.

We have predicted the correlation among  $A_{\text{CP}}^{b \rightarrow s\gamma}$  and  $\mathcal{S}_f$  of the  $B_d^0$  and  $B_s^0$  decays. These CP-violating asymmetries could deviate from the SM predictions.

Since we suppose rather small  $\mu \tan \beta$ ,  $\mathcal{O}(1 \text{ TeV})$ , the contribution from  $LL(RR)$  components are minor in these CP-violating asymmetries. In this case, the new physics contribution is minor in  $M_{12}^q$  of  $B_q - \bar{B}_q$  ( $q = d, s$ ) mixing since  $|(\delta_d^{LR})_{23}|$  is at most  $\mathcal{O}(10^{-3})$ . This result is consistent with the recent result of the CP violations at LHCb as discussed in Section 2.

In the near future, the precise data of the direct CP violation of  $b \rightarrow s\gamma$  and CP-violating asymmetries in the non-leptonic decays of  $B_d^0$  and  $B_s^0$  mesons give us the crucial test for our framework of the squark flavor mixing.

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